First Name: ______________________ Last Name: ______________________

Student-No: ______________________ Section: ______________________

Very short answer questions

1. 2 marks Each part is worth 1 mark. Please write your answers in the boxes. **Marking scheme:** 1 for each correct, 0 otherwise

   (a) Find the domain of continuity for the function $f(x) = \log\big(4x^2 - 1\big)$.

   Answer: $(-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, +\infty)$

   Solution: The function is continuous when $4x^2 - 1 > 0$, i.e. $(2x-1)(2x+1) > 0$, which yields the intervals $(-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, +\infty)$.

   (b) Compute $\lim_{t \to 1} \sqrt{5x^3 + 4}$.

   Answer: 3

   Solution:

$$\lim_{t \to 1} \sqrt{5x^3 + 4} = \sqrt{\lim_{t \to 1}\big(5x^3 + 4\big)} = \sqrt{5 \lim_{t \to 1}(x^3) + 4} = \sqrt{9} = 3.$$

Short answer questions — you must show your work

2. 4 marks Each part is worth 2 marks.

   (a) Compute the limit $\lim_{x \to -3} \dfrac{x^2 - 9}{x + 3}$

   Solution: If try naively then we get $0/0$, so we simplify first:

$$\frac{x^2 - 9}{x + 3} = \frac{(x-3)(x+3)}{(x+3)} = x - 3$$

   Hence the limit is $\lim_{x \to -3}(x - 3) = -6$. **Marking scheme:** 1 for factoring+cancelling, 1 for answer. If answer with no working then 0.

   (b) Find all values of c such that the following function is continuous:

$$f(x) = \begin{cases} x^2 + c & \text{if } x < c \\ 2cx - 2 & \text{if } x \geq c \end{cases}$$

Solution: The function is continuous for $x \neq c$ since each of those two branches are polynomials. So, the only question is whether the function is continuous at $x = c$; for this we need

$$\lim_{x\to c^-} f(x) = f(c) = \lim_{x\to c+} f(x).$$

We compute

$$\begin{aligned}\lim_{x\to c^-} f(x) &= \lim_{x\to c^-} x^2 + c = c^2 + c;\\ f(c) &= 2c \cdot c - 2 = 2c^2 - 2;\\ \lim_{x\to c^+} f(x) &= \lim_{x\to c^+} 2cx - 2 = 2c^2 - 2.\end{aligned}$$

So, we need $c^2 + c = 2c^2 - 2$, which yields $c^2 - c - 2 = (c-2)(c+1) = 0$, i.e. $c = -1$ or $c = 2$.

Marking scheme:

- 1 mark for writing the condition for continuity $\lim_{x\to c^-} f(x) = f(c) = \lim_{x\to c+} f(x)$.
- 1 mark for solving correctly and finding **both** solutions $c = -1$ and $c = 2$.

Long answer question — you must show your work

3. 4 marks Compute the limit $\lim_{x\to 1} \dfrac{\sqrt{x+2} - \sqrt{4-x}}{x-1}$.

Solution: If we try to do the limit naively we get $0/0$. Hence we must simplify. **Marking scheme:** If multiply by correct conjugate then 2 marks. If multiply by something close to correct then 1. Else 0.

$$\begin{aligned}\frac{\sqrt{x+2} - \sqrt{4-x}}{x-1} &= \frac{\sqrt{x+2} - \sqrt{4-x}}{x-1} \cdot \frac{\sqrt{x+2} + \sqrt{4-x}}{\sqrt{x+2} + \sqrt{4-x}}\\ &= \frac{(x+2) - (4-x)}{(x-1)(\sqrt{x+2} + \sqrt{4-x})}\\ &= \frac{2x-2}{(x-1)(\sqrt{x+2} + \sqrt{4-x})}\\ &= \frac{2}{\sqrt{x+2} + \sqrt{4-x}}\end{aligned}$$

Marking scheme: If correct simplification then 1 mark, else 0. So the limit is

$$\begin{aligned}\lim_{x\to 1} \frac{\sqrt{x+2} - \sqrt{4-x}}{x-1} &= \lim_{x\to 1} \frac{2}{\sqrt{x+2} + \sqrt{4-x}}\\ &= \frac{2}{\sqrt{3} + \sqrt{3}}\\ &= \frac{1}{\sqrt{3}}\end{aligned}$$

Marking scheme: 1 for answer.